**Bottleneck Lesson Plan—Degrees of Freedom**

1. The bottleneck—What are students unable to do? Students find it difficult to understand that during the actual data collection process all scores are free to vary and the concept of degrees of freedom does not apply. Degrees of freedom only come into play after the data have been collected and we are calculating statistics on those data.
2. Mental Action—What mental actions does the expert perform to get past the bottleneck?

Statistically, the *df* are the number of scores that are free to vary when calculating a statistic, or in other words, the number of pieces of independent information available when calculating a statistic.

1. Model the thinking—What analogy will you use to model these mental actions?

Imagine you’re a fun-loving person who loves to wear hats. You believe that variety is the spice of life. Unfortunately, you only 7 hats. Yet you want to wear a different hat every day of the week.



On the first day, you can wear any of the 7 hats. On the second day, you can choose from the 6 remaining hats, on day 3 you can choose from 5 hats, and so on.

When day 6 rolls around, you still have a choice between 2 hats that you haven’t worn yet that week. But after you choose your hat for day 6, you have no choice for the hat that you wear on Day 7. You must wear the one remaining hat. You had 7-1 = 6 days of “hat” freedom—in which the hat you wore could vary!

That’s kind of the idea behind degrees of freedom in statistics. Degrees of freedom are often broadly defined as the number of "observations" (pieces of information) in the data that are free to vary when estimating statistical parameters. (from <https://blog.minitab.com/en/statistics-and-quality-data-analysis/what-are-degrees-of-freedom-in-statistics>)

Example from Psychology: Suppose you are told that a student took three quizzes, each worth a total of 10 points. You are asked to guess what her scores were. In this scenario, you may guess any three numbers as long as they are in the range from 0 to 10. In this example, you have 3 *df*, for each score is free to vary. Each score is an independent piece of information. Choosing the score for one quiz has no effect on either of the other two scores that you may choose.

But now I give you some information about the student’s performance by telling you that the total of her scores was 27. I have now created a scenario with 2 *df.*Suppose you guess 10 for the first score. Does choosing this score place any limitation on what you might guess for a score on the second, given that the total of the scores must be 27? No, for your choice of a second score is still free to vary from 0 to 10. You guess 9 for a second score. What about your choice of a third score? What must it be. If the total of the three scores is 27, and the first score you chose was 10, and the second 9, then your third choice must be 8 for a total of 27 to be obtained. In this instance, the third score is not free to vary if you know the total of the scores and any two of the three scores. For this example then, there are 2 *df* in the choice of scores. If you know the total of the three scores, then only two provide independent information, the third score becomes dependent on previous two scores. By giving you knowledge of the total of the scores I have reduced the *df*in the number of choices you have.

(from https://statisticalsage.wordpress.com/2011/09/06/difficult-concepts—degrees-of-freedom/)

1. Practice and Feedback—How will the students practice these mental actions? How will they receive feedback to make improvements?

03 individual df practice document:: Is it a sample or a population?

1. Tim wonders why the Maple trees in Erie have black spots on their leaves. He is only interested in the Maple trees in Erie. Is this a sample or a population?

Will Tim use N or n-1? Why did you choose this one?

2. Vanessa is interested in how the tomatoes in her garden compares to the tomatoes in all of Erie. She wants to compare the average weights of the tomatoes. Is this a sample or population?

Will Vanessa use N or n-1? Why did you choose this one?

3. Gary is interested in the number of Cokes that students in his statistics class drink while studying compared to the number of Cokes that all the students in all statistics courses drink while studying. Is this a sample or a population?

Will Gary use N or n-1? Why did you choose this one?

4. Wesley is interested if people in Erie have more cars than people in the U.S. in general. Is this a sample or a population?

Will Wesley use N or n-1? Why did you choose this one?

5. Joan is interested in how smart Chihuahuas in Erie are and how many tricks they know. Is this a sample or a population?

Will Joan use N or n-1? Why did you choose this one?

1. Motivation—What will I do to hold students accountable and disrupt ritual ways of learning?

I will use encouragement by walking around the room and saying positive things about their performance in class.

1. Assessment—How will I assess student mastery of the mental actions?

Exam at the end of the module